

Multipartite entanglement for open system in noninertial frames

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Abstract

Based on Greenberger-Horne-Zeilinger (*GHZ*) and *W* initial states, the tripartite entanglement of a fermionic system under the amplitude damping channel and in depolarizing noise when two subsystems accelerated is investigated. Unlike the case of two-qubit system in which sudden death occurs easily, we find here that the sudden death never occurs even all subsystems are under the noise environment. We note that both acceleration and environment can destroy the symmetry between the subsystems, but the effect of environment is much stronger than that of acceleration. We also show that an entanglement rebound process will take place when $P > 0.75$ in the depolarizing noise and the larger acceleration will result in the weaker rebound process.

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I. INTRODUCTION

Quantum entanglement plays as an important resource in quantum computation [1], teleportation [2], dense coding [3] and cryptography [4, 5]. Since the environment is unavoidable in practice, there were many meaningful works on the dynamics of entanglement in two qubits which interacted with different kinds of environments, and some important features of the entanglement such as the entanglement sudden death [6–9] and birth [10] were found. On the other hand, the relativistic quantum information has been a focus of research over recent years for both conceptual and experimental reasons. The studies of entanglement in noninertial frames have shown that the Unruh or Hawking effect will influence the degree of entanglement [11–15] dramatically. However, most of these works focused on the study of quantum information in bipartite systems and only one of the subsystems accelerated.

Along this line, in real quantum information tasks we have to consider multipartite entanglement which interacts with different kind of environments in inertial or noninertial frame. Recently, the tripartite entanglement of scalar and Dirac fields in noninertial frames were studied by Mi-Ra Hwang *et al.* [16] and Jieci Wang *et al.* [17]. They showed that the tripartite entanglement decreases with the increase of the acceleration, and all the two-tangles equal to zero when one or two observers accelerated for *GHZ* initial state which is exactly the same as the two-tangles obtained in the inertial frame.

In this paper we will discuss the tripartite entanglement of Dirac fields under the environment for amplitude damping channel and depolarizing noise when two observers accelerated for *GHZ* and *W* initial states. Our setting consists of three observers: Alice, Bob and Charlie. We assume that Alice is in an inertial frame, while Bob and Charlie are in accelerated frames with the same uniformly acceleration. We will focus our attention on how the environment and acceleration influence the degree of tripartite entanglement. We first let the inertial observer Alice under the environment, then the noninertial observer Charlie under the environment, and at last all of them under the environment. The Dirac fields, as shown in Refs. [18–20], from an inertial perspective, can be described as a superposition of Minkowski monochromatic modes $|0\rangle_M = \bigotimes_i |0_{\omega_i}\rangle_M$ and $|1\rangle_M = \bigotimes_i |1_{\omega_i}\rangle_M \forall i$, with

$$\begin{aligned} |0_{\omega_i}\rangle_M &= \cos r_i |0_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} + \sin r_i |1_{\omega_i}\rangle_I |1_{\omega_i}\rangle_{II}, \\ |1_{\omega_i}\rangle_M &= |1_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II}, \end{aligned} \tag{1}$$

where $\cos r_i = (e^{-2\pi\omega_i c/a_i} + 1)^{-1/2}$, a_i is the acceleration of the observer i and c is the acceleration of the light. On account of the accelerated observer in Rindler region I are causally disconnected from region II , by tracing over the inaccessible modes we will obtain a tripartite state and then we can calculate tripartite entanglement of the 3-qubit states.

This paper is structured as follows. In Sec. II we will study the environment and some measurements of tripartite entanglement. In Sec. III we will discuss the tripartite entanglement of Dirac fields when two observers are accelerated for GHZ state and compare the case of different subsystems under the environment. In Sec. IV we will analyze the tripartite entanglement for the W state under the environment. Our work will be summarized in the last section.

II. THE ENVIRONMENT AND MEASURES

Here we consider the local channel, in which all the subsystems interact independently with its own environment and no communication appears. If the local environment acts independently on subsystem's state, the total evolution of these qubit systems can be expressed as [21]

$$L(\rho_s) = \sum_{\mu \dots \nu} M_\mu^1 \otimes \dots \otimes M_\nu^N \rho_s M_\mu^{1\dagger} \otimes \dots \otimes M_\nu^{N\dagger}, \quad (2)$$

where M_μ^i are the Kraus operators and N is the number of the subsystems.

For the amplitude damping environment, we can take

$$M_0^i = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-P_i} \end{pmatrix}, \quad M_1^i = \begin{pmatrix} 0 & \sqrt{P_i} \\ 0 & 0 \end{pmatrix}, \quad (3)$$

where $i = (1, 2 \dots N)$, $\mu = (0, 1)$. This channel represents the dissipative interaction between the qubit and its environment. The emblematic example is given by the spontaneous emission of a photon by a two-level atom into a zero-temperature environment of electromagnetic-field modes. A simple way to gain insight about this process is through the corresponding quantum map [21]

$$|0\rangle_S |0\rangle_E \rightarrow |0\rangle_S |0\rangle_E, \quad (4)$$

$$|1\rangle_S |0\rangle_E \rightarrow \sqrt{1-P} |1\rangle_S |0\rangle_E + \sqrt{P} |0\rangle_S |1\rangle_E. \quad (5)$$

Eq. (4) shows that, if the system stays at $|0\rangle_S$, both it and its environment will not change at all. Eq. (5) indicates that, if the system stays at $|1\rangle_S$, the decay will exist in the system with probability P , and it can also remain there with probability $(1 - P)$.

For the depolarizing noise, due to the state is not stable absolutely in this channel, the qubits will have three mistakenly flip in random. Assume that the three mistakenly flip take the same probability then the responding quantum map becomes

$$|\Psi\rangle_S|0\rangle_E \rightarrow \sqrt{1-P_i}|\Psi\rangle_S|0\rangle_E + \sqrt{\frac{P_i}{3}}|\Psi\rangle_S|1\rangle_E + \sqrt{\frac{P_i}{3}}|\Psi\rangle_S|2\rangle_E + \sqrt{\frac{P_i}{3}}|\Psi\rangle_S|3\rangle_E, \quad (6)$$

and now the Kraus operators are

$$M_0^i = \sqrt{1-P_i}\sigma_0, \quad M_1^i = \sqrt{\frac{P_i}{3}}\sigma_1, \quad M_2^i = \sqrt{\frac{P_i}{3}}\sigma_2, \quad M_3^i = \sqrt{\frac{P_i}{3}}\sigma_3, \quad (7)$$

where σ_μ are the Pauli operators, $i = (1, 2 \cdots N)$ and $\mu = (0, 1, 2, 3)$.

For both two environments P_i is the decay parameter relating only to time. Under the Markov approximation, the relationship between the parameter P_i and the time t is shown by $P_i = (1 - e^{-\Gamma_i t})$ [21, 22], here Γ_i is the decay rate.

On the other hand, the negativity is used to measure a bipartite system ρ_{AB} , which is defined as [23]

$$N_{AB} = \|\rho_{AB}^{T_\alpha}\| - 1, \quad (8)$$

where T_α denotes the partial transpose of ρ_{AB} and $\|\cdot\|$ is the trace norm of a matrix. For any 3-qubit states $|\Phi\rangle_{ABC}$, N_{AB} is two-tangle which is the negativity of the mixed state $\rho_{AB} = \text{Tr}_C(|\Phi\rangle_{ABC}\langle\Phi|)$, and $N_{A(BC)}$ is one-tangle which is defined as

$$N_{A(BC)} = \|\rho_{ABC}^{T_\alpha}\| - 1. \quad (9)$$

Then the so-called residual entanglement becomes

$$\pi_A = N_{A(BC)}^2 - N_{AB}^2 - N_{AC}^2, \quad (10)$$

and the π -tangle π_{ABC} is defined as

$$\pi_{ABC} = \frac{1}{3}(\pi_A + \pi_B + \pi_C), \quad (11)$$

which describes an average residual entanglement.

III. TRIPARTITE ENTANGLEMENT FOR GHZ INITIAL STATE UNDER ENVIRONMENT

We assume that Alice, Bob and Charlie share a GHZ initial state

$$|\Phi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0_{\omega_a}\rangle_A|0_{\omega_b}\rangle_B|0_{\omega_c}\rangle_C + |1_{\omega_a}\rangle_A|1_{\omega_b}\rangle_B|1_{\omega_c}\rangle_C), \quad (12)$$

where $|0_{\omega_{a(b,c)}}\rangle_{A(B,C)}$ and $|1_{\omega_{a(b,c)}}\rangle_{A(B,C)}$ are vacuum states and the first excited states from the perspective of an inertial observer. We also assume that Alice, Bob and Charlie each carry a monochromatic detector sensitive to frequencies ω_a , ω_b and ω_c , respectively. Using Eq. (1) and tracing over the disconnected region II , we can get the state in Rindler spacetime

$$|\Phi\rangle_{AB_I C_I} = \frac{1}{\sqrt{2}}[\cos r_b \cos r_c |0\rangle_A |0\rangle_{B_I} |0\rangle_{C_I} + \cos r_b \sin r_c |0\rangle_A |0\rangle_{B_I} |1\rangle_{C_I} + \sin r_b \cos r_c |0\rangle_A |1\rangle_{B_I} |0\rangle_{C_I} + \sin r_b \sin r_c |0\rangle_A |1\rangle_{B_I} |1\rangle_{C_I} + |1\rangle_A |1\rangle_{B_I} |1\rangle_{C_I}], \quad (13)$$

hereafter frequency subscripts are dropped. Then we obtain the density matrix

$$\begin{aligned} \rho_{AB_I C_I} = & \frac{1}{2}[\cos^2 r_b \cos^2 r_c |000\rangle\langle 000| + \cos^2 r_b \sin^2 r_c |001\rangle\langle 001| \\ & + \sin^2 r_b \cos^2 r_c |010\rangle\langle 010| + \sin^2 r_b \sin^2 r_c |011\rangle\langle 011| \\ & + \cos r_b \cos r_c (|000\rangle\langle 111| + |111\rangle\langle 000|) + |111\rangle\langle 111|], \end{aligned} \quad (14)$$

where $|mnl\rangle = |m\rangle_A |n\rangle_{B_I} |l\rangle_{C_I}$. For simplification, in what follows we just consider the case that both Bob and Charlie move with the same acceleration, i. e. $r_b = r_c = r$.

A. Amplitude damping channel

Now we let all the subsystems interact with amplitude damping environment. Using Eqs. (2) and (3), we get the evolved state

$$\begin{aligned} \rho_{AB_I C_I}^{evo} = & \frac{1}{2}\{[\cos^4 r + (n+m)\cos^2 r \sin^2 r + mn \sin^4 r + pmn]|000\rangle\langle 000| \\ & + [(1-n)(\cos^2 r \sin^2 r + m \sin^4 r + pm)]|001\rangle\langle 001| + (1-p)(1-m)n|110\rangle\langle 110| \\ & + [(1-m)(\cos^2 r \sin^2 r + n \sin^4 r + pn)]|010\rangle\langle 010| + (1-p)(1-n)m|101\rangle\langle 101| \\ & + [(1-m)(1-n)(\sin^4 r + p)]|011\rangle\langle 011| + (1-p)mn|100\rangle\langle 100| \\ & + \sqrt{(1-p)(1-m)(1-n)} \cos^2 r (|000\rangle\langle 111| + |111\rangle\langle 000|) \\ & + (1-p)(1-m)(1-n)|111\rangle\langle 111|\}, \end{aligned} \quad (15)$$

where p , m , n are the decay probability for Alice, Bob and Charlie, respectively. After some calculations we find one-tangles

$$\begin{aligned} N_{A(B_I C_I)} = & \frac{1}{2}\{-1 - mn - p + mp + np + \cos^4 r + 2 \cos^2 r \sin^2 r + m \sin^4 r + n \sin^4 r - mn \sin^4 r \\ & + \sqrt{(-1+p)[m^2 n^2 (-1+p) - (-1+m)(-1+n) \cos^4 r]} \\ & + \sqrt{(-1+m)(-1+n)[(-1+p) \cos^4 r + (-1+m)(-1+n)(p + \sin^4 r)^2]}\}. \end{aligned} \quad (16)$$

$$N_{B_I(AC_I)} = \frac{1}{2} \{ \sqrt{(-1+m)(-(-1+p)(-1+n) \cos^4 r + (-1+m)[\cos^2 r \sin^2 r + n(p + \sin^4 r)]^2)} \\ + \sqrt{(-1+n)(-1+p)[m^2(-1+n)(-1+p) - (-1+m) \cos^4 r] + \sin^4 r + mn \sin^4 r} \\ -1 - m + mn + mp - np + \cos^4 r + \cos^2 r \sin^2 r + m \cos^2 r \sin^2 r - n \sin^4 r \}, \quad (17)$$

$$N_{C_I(AB_I)} = \frac{1}{2} \{ \sqrt{(-1+n)(-(-1+p)(-1+m) \cos^4 r + (-1+n)[\cos^2 r \sin^2 r + m(p + \sin^4 r)]^2)} \\ + \sqrt{(-1+m)(-1+p)[n^2(-1+m)(-1+p) - (-1+n) \cos^4 r] + \sin^4 r + mn \sin^4 r} \\ -1 - n + mn - mp + np + \cos^4 r + \cos^2 r \sin^2 r + n \cos^2 r \sin^2 r - m \sin^4 r \}. \quad (18)$$

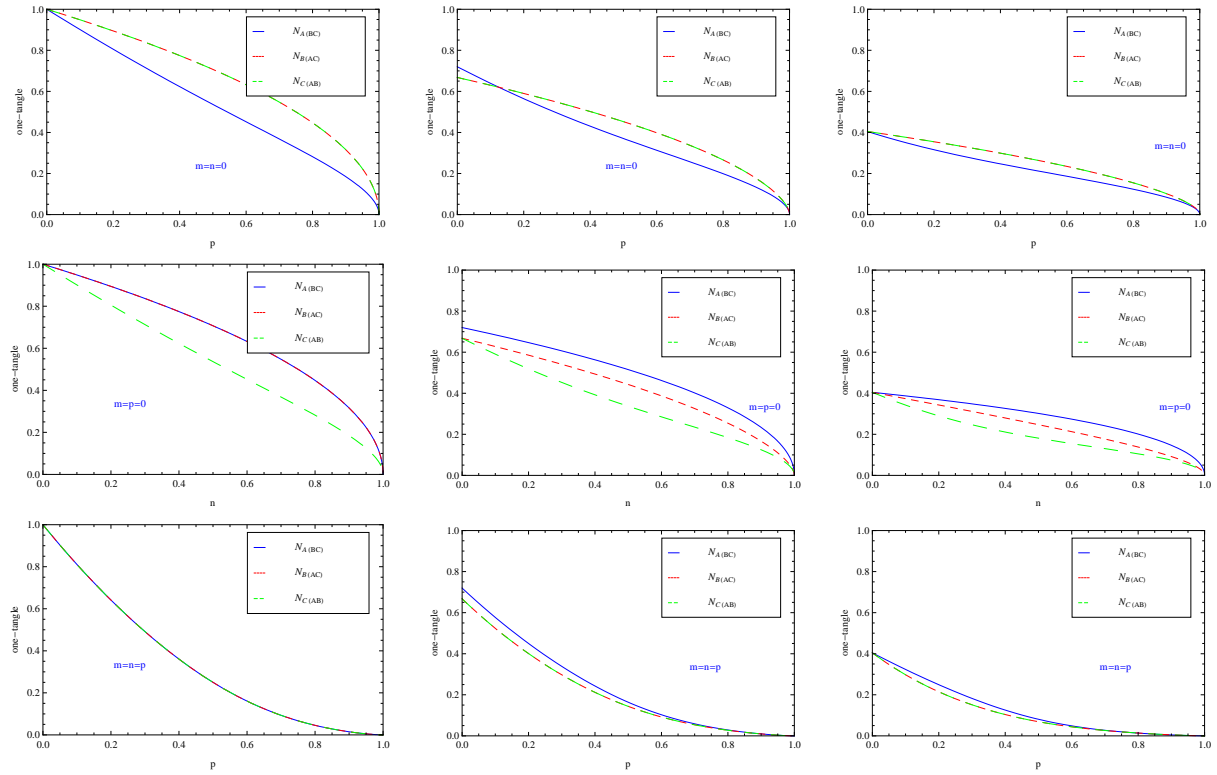


FIG. 1: (Color online) The plot shows the negativity $N_{A(B_I C_I)}$ (blue line), $N_{B_I(AC_I)}$ (red line) and $N_{C_I(AB_I)}$ (green line) for amplitude damping channel. The first (second) row presents that only inertial observer Alice (noninertial observer Charlie) is under the environment. And the third row is for the case that Alice, Bob, and Charlie all interact with the environment. We draw them for $r = 0$ (left rank), $r = \pi/6$ (middle rank), and $r = \pi/4$ (right rank).

One-tangles are shown by the first row in Fig.1 with $m = n = 0$, which means that only the inertial observer Alice interacts with environment. All the one-tangles decrease as the interaction increases, and disappear completely when $p = 1$ which means the entire tripartite entanglement

is destroyed at an infinite time. Note that $N_{B_I(AC_I)} = N_{C_I(AB_I)}$ for any time which indicates Bob's and Charlie's subsystem are symmetry when only Alice acts with the environment. The intersect point in the middle picture indicates that $N_{A(B_I C_I)} = N_{B_I(AC_I)} = N_{C_I(AB_I)}$, which means that there is no difference among all the subsystems at this point. Generally, the intersect point is

$$p = \cos 2r \sin^2 r, \quad (19)$$

which means that when p and r satisfy this relationship we can't distinguish the three subsystems by the negativity. We also see that a bigger acceleration means a smaller initial entanglement for one-tangle as expected. When $r = \pi/4$ the three one-tangles have the same initial entanglement which is the same as the result in [17]. It is worth to note that the sudden death never occurs for one-tangle even Bob and Charlie are in the limit of infinite acceleration.

We show the one-tangles by second row in Fig. 1 with $p = m = 0$, which means that only the noninertial observer Charlie is under the amplitude damping environment. Obviously, if $r = 0$ the decay curve would be the same with the former case with $r = 0$. However, if $r \neq 0$ we see that $N_{B_I(AC_I)}$ decreases more slowly than $N_{C_I(AB_I)}$ because Bob doesn't interact with environment while Charlie does, and $N_{A(B_I C_I)}$ decays more slowly than $N_{B_I(AC_I)}$ because Bob has an acceleration while Alice doesn't. That is to say, both the acceleration and environment can destroy the symmetry between the subsystems, which can be used to distinguish them. It is interesting to note that no intersect point and no sudden death exist in this case.

The situation with $m = n = p$ is shown by the third row in Fig. 1, which means Alice, Bob, and Charlie are all under the same environment. It is easy to find out that all the one-tangles decrease more quickly than the former two cases, which is similar to the behaviors of bipartite entanglement. Especially, if $r = 0$ we find all the three subsystems have the same decay curve due to they are highly symmetric and indistinguishable. And if $r \neq 0$, $N_{B_I(AC_I)} = N_{C_I(AB_I)}$ for all the time because of their symmetry again. Even all the subsystems interact with the environment and in the infinite acceleration there is still no sudden death yet.

On the other hand, by use of Eq. (8) we compute the two-tangle between any two subsystems of the multipartite system. Tracing the qubit of C_I we get $N_{AB_I} = 0$, which means that no bipartite entanglement exist between A and B_I with considering of both environment and acceleration. Similarly, it is easy to obtain $N_{AC_I} = N_{B_I C_I} = 0$.

In addition, we calculate the π -tangle by use of Eqs. (10) and (11)

$$\pi_{AB_I C_I} = \frac{1}{3}(\pi_A + \pi_{B_I} + \pi_{C_I}) = \frac{1}{3}[N_{A(B_I C_I)}^2 + N_{B_I(AC_I)}^2 + N_{C_I(AB_I)}^2]. \quad (20)$$

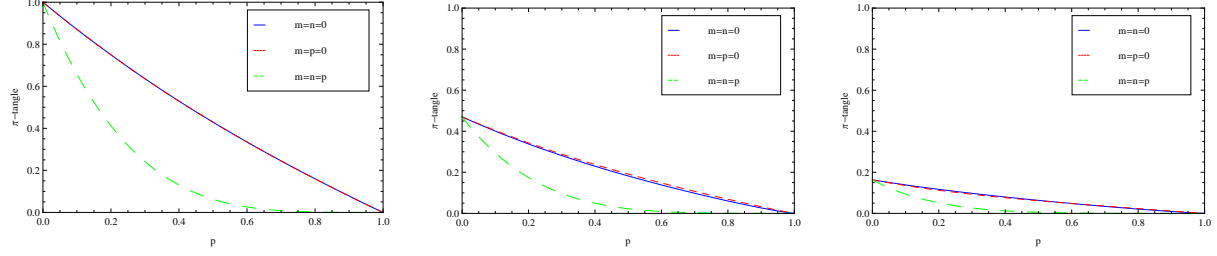


FIG. 2: (Color online) Blue (red) line plots the π -tangle for the case that only Alice (Charlie) interacts with the environment. Green line plots the case of all of them interacting with the environment. We show three cases for $r = 0$ (left), $r = \pi/6$ (middle), and $r = \pi/4$ (right). It is worth to note that there is still no sudden death.

We give the results in Fig. 2 for the former three cases. Note that a bigger acceleration also means a smaller initial π -tangle just like before. It is interesting to note that the π -tangle decay curves for $m = n = 0$ and $m = p = 0$ are almost the same, which indicates that the effect of environment is so strong that we can nearly ignore the effect of acceleration for tripartite entanglement. We also find that the more strong subsystems interact with environment, the more quickly the π -tangle decreases. We can prove that there is still no sudden death yet. Taking the highest possible case when $m = n = p$, $r = \pi/4$, we have

$$\pi_{AB|C_I} = \frac{1}{192} \{ 2[1 + 4p - 2\sqrt{(p-1)^3(-1-4p^2+4p^3)} - (1-p)\sqrt{5-2p+9p^2+8p^3+16p^4}]^2 + [3p^2 - 1 - 2p + \sqrt{(p-1)^3(-5-7p-8p^2+16p^3)} - 2(p-1)\sqrt{1-p+4p^4}]^2 - 5p^2 \}. \quad (21)$$

It is easy for us to find that $\pi_{AB|C_I}$ is monotone decreasing function when $0 < p < 1$ and it exactly equals to zero when $p = 1$ which indicates that no sudden death appears.

B. Depolarizing noise

We present the results in Fig. 3 for the case that all the subsystems are in depolarizing noise. We see that many characteristics for amplitude damping channel still remain under this environment. But the one-tangle in depolarizing noise decays much more quickly than it goes under amplitude damping channel. What's surprising is that the one-tangles decays to zero at $p = 0.75$ and then a rebound process takes place when $p > 0.75$. This means that all the tripartite entanglement transfers to environment at $p = 0.75$ and then part of it transfers from environment back to the system when $p > 0.75$. The bigger the acceleration is, the smaller this rebound process becomes.

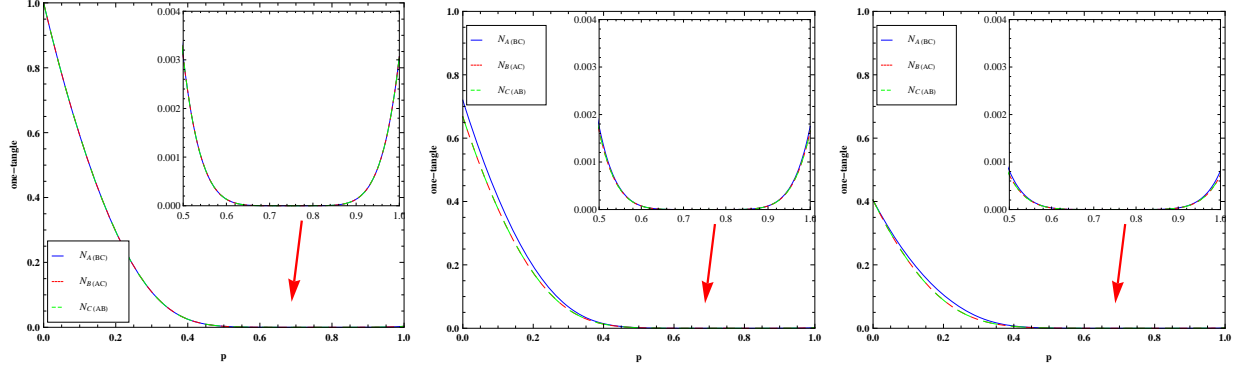


FIG. 3: (Color online) The negativity $N_{A(B|C_I)}$ (blue line), $N_{B|AC_I}$ (red line), and $N_{C_I|AB_I}$ (green line) when all Alice, Bob, and Charlie are in depolarizing noise. We show three cases for $r = 0$ (left), $r = \pi/6$ (middle), and $r = \pi/4$ (right). The rebound process for entanglement is plotted in the magnifying pictures.

In addition, using Eq. (8) we get again that $N_{AB_I} = N_{AC_I} = N_{B_I C_I} = 0$, which is exactly the same as before. That is to say, either in amplitude damping channel or in depolarizing noise, either in inertial frame or in noninertial frame, there is no bipartite entanglement in this system for *GHZ* state. We also get the π -angle by use of Eqs. (10) and (11), then plot it and its rebound process in Fig.4 .

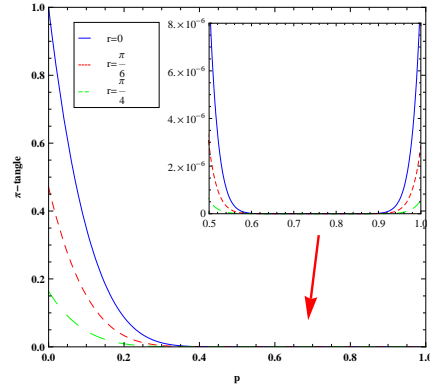


FIG. 4: (Color online) The π -angle $\pi_{AB_I C_I}$ when Alice, Bob, and Charlie are all in depolarizing noise. We plot three cases for $r = 0$ (blue line), $r = \pi/6$ (red line), and $r = \pi/4$ (green line). The rebound process for entanglement is plotted in the magnifying picture.

At last, we note that the CKW inequality [24], $N_{AB}^2 + N_{AC}^2 \leq N_{A(BC)}^2$, is saturated for this initial state, which means the effect of both environment and acceleration doesn't destroy this inequality for *GHZ* initial state.

IV. TRIPARTITE ENTANGLEMENT FOR W STATE UNDER THE ENVIRONMENT

Now we assume Alice, Bob and Charlie share a W initial state

$$|\Phi\rangle_{ABC} = \frac{1}{\sqrt{3}}(|0\rangle_A|0\rangle_B|1\rangle_C + |0\rangle_A|1\rangle_B|0\rangle_C + |1\rangle_A|0\rangle_B|0\rangle_C). \quad (22)$$

With the help of Eq. (1) we obtain the system's density matrix

$$\begin{aligned} \rho_{ABIC_I} = & \frac{1}{3}[\cos^2 r_b|001\rangle\langle 001| + \cos^2 r_c|010\rangle\langle 010| + (\sin^2 r_b + \sin^2 r_c)|011\rangle\langle 011| \\ & + \cos^2 r_b \cos^2 r_c|100\rangle\langle 100| + \cos^2 r_b \sin^2 r_c|101\rangle\langle 101| + \cos^2 r_c \sin^2 r_b|110\rangle\langle 110| \\ & + \sin^2 r_b \sin^2 r_c|111\rangle\langle 111| + \cos r_b \cos r_c(|010\rangle\langle 001| + |001\rangle\langle 011|) \\ & + \cos^2 r_b \cos r_c(|100\rangle\langle 001| + |001\rangle\langle 100|) + \cos r_b \cos^2 r_c(|100\rangle\langle 010| + |010\rangle\langle 100|) \\ & + \cos r_b \sin^2 r_c(|101\rangle\langle 011| + |011\rangle\langle 101|) + \cos r_c \sin^2 r_b(|110\rangle\langle 011| + |011\rangle\langle 110|)]. \end{aligned} \quad (23)$$

A. Amplitude damping channel

Here we also just consider Bob and Charlie move with the same acceleration, i. e. $r_b = r_c = r$. In amplitude damping channel, by use of Eqs. (2) and (3), then we get the evolved state

$$\begin{aligned} \rho_{ABIC_I}^{evo} = & \frac{1}{3}\{[(m+n)\cos^2 r(1+p\sin^2 r) + p\cos^4 r + mn\sin^2 r(2+p\sin^2 r)]|000\rangle\langle 000| \\ & + \frac{1}{8}(1-n)[4+8m+p+3mp-4(-1+m(2+p))\cos 2r + (-1+m)p\cos 4r]|001\rangle\langle 001| \\ & + \frac{1}{8}(1-m)[4+8n+p+3np-4(-1+n(2+p))\cos 2r + (-1+n)p\cos 4r]|010\rangle\langle 010| \\ & - \frac{1}{2}(-1+m)(-1+n)(-4-p+p\cos 2r)\sin^2 r|011\rangle\langle 011| \\ & - \frac{1}{4}(-1+p)(-1-m+(-1+m)\cos 2r)(-1-n+(-1+n)\cos 2r)|100\rangle\langle 100| \\ & - \frac{1}{2}(-1+n)(-1+p)(-1-m+(-1+m)\cos 2r)\sin^2 r|101\rangle\langle 101| \\ & - \frac{1}{2}(-1+m)(-1+p)(-1-n+(-1+n)\cos 2r)\sin^2 r|110\rangle\langle 110| \\ & - (-1+m)(-1+n)(-1+p)\sin^4 r|111\rangle\langle 111| \\ & + \sqrt{1-m}\sqrt{1-n}\cos^2 r|001\rangle\langle 010| + \sqrt{1-m}\sqrt{1-n}\cos^2 r|010\rangle\langle 001| \\ & + \sqrt{(-1+n)(-1+p)}\cos r(\cos^2 r + m\sin^2 r)(|100\rangle\langle 001| + |001\rangle\langle 100|) \\ & + \sqrt{(-1+m)(-1+p)}\cos r(\cos^2 r + n\sin^2 r)(|100\rangle\langle 010| + |010\rangle\langle 100|) \\ & + \sqrt{(-1+m)(-1+n)^2(-1+p)}\cos r\sin^2 r(|101\rangle\langle 011| + |011\rangle\langle 101|) \\ & + \sqrt{(-1+m)^2(-1+n)(-1+p)}\cos r\sin^2 r(|110\rangle\langle 011| + |011\rangle\langle 110|)\}, \end{aligned} \quad (24)$$

where p, m, n are the decay probability when Alice, Bob and Charlie interact with amplitude damping environment, respectively. Then one-tangles are given by

$$\begin{aligned}
N_{A(B_I C_I)} = & \frac{1}{6} \{-6 - \beta\gamma(-4 - p + p \cos 2r) \sin^2 r - 2\alpha\zeta\eta + \sqrt{\beta\alpha[\beta\alpha(1 + n - \gamma \cos 2r)^2 \sin^4 r + 4\epsilon]} \\
& + 2\sqrt{\beta\gamma\alpha \sin^4 r [(-2 + m + n) \cos^2 r + \beta\gamma\alpha \sin^4 r]} \\
& + 2\sqrt{\gamma(\beta \cos^4 r + \beta\gamma\alpha \cos^2 r \sin^4 r + \gamma[\cos^2 r(1 + p \sin^2 r) + m\tau \sin^2 r]^2)} \\
& + 2\sqrt{\beta(\gamma \cos^4 r + \beta\gamma\alpha \cos^2 r \sin^4 r + \beta[\cos^2 r(1 + p \sin^2 r) + n\tau \sin^2 r]^2)} \\
& + 2\sqrt{\gamma\alpha\delta + \beta\alpha\epsilon + \epsilon} + \sqrt{\gamma\alpha[\gamma\alpha(1 + m - \beta \cos 2r)^2 \sin^4 r + 4\delta]\}, \tag{25}
\end{aligned}$$

$$\begin{aligned}
N_{B(AC_I)} = & \frac{1}{6} \{-6 - \alpha\gamma(-1 - m + \beta \cos 2r) \sin^2 r - 2\gamma\sqrt{\beta\alpha \sin^4 r (\cos^2 r + \alpha\beta \sin^4 r)} + \\
& 2\sqrt{\beta\gamma \cos^4 r + \alpha\beta\epsilon + [p \cos^4 r + (m + n) \cos^2 r(1 + p \sin^2 r) + mn\tau \sin^2 r]^2} \\
& + \sqrt{\beta\gamma[4 \cos^4 r + 4\alpha\beta \cos^2 r \sin^4 r + \beta\gamma(4 + p - p \cos 2r)^2]} + 2\sqrt{\alpha(\alpha\zeta^2\eta^2 + \gamma\delta)} \\
& + \sqrt{\alpha\beta[4\beta\gamma \cos^2 r \sin^4 r + \alpha\beta(1 + n - \gamma \cos 2r)^2 \sin^4 r + 4\epsilon]} \\
& + 2\sqrt{\gamma\{\alpha\beta\gamma \cos^2 r \sin^4 r + \alpha\delta + \gamma[\cos^2 r(1 + p \sin^2 r) + m\tau \sin^2 r]^2\}} \\
& - 2\beta[\cos^2 r(1 + p \sin^2 r) + n \sin^2 r(2 + p \sin^2 r)]\}, \tag{26}
\end{aligned}$$

$$\begin{aligned}
N_{C(AB_I)} = & \frac{1}{6} \{-6 - \alpha\beta(-1 - n + \gamma \cos 2r) \sin^2 r - 2\beta\sqrt{\gamma\alpha \sin^4 r (\cos^2 r + \alpha\gamma \sin^4 r)} + \\
& 2\sqrt{\beta\gamma \cos^4 r + \alpha\gamma\delta + [p \cos^4 r + (m + n) \cos^2 r(1 + p \sin^2 r) + mn\tau \sin^2 r]^2} \\
& + \sqrt{\beta\gamma[4 \cos^4 r + 4\alpha\gamma \cos^2 r \sin^4 r + \beta\gamma(4 + p - p \cos 2r)^2]} + 2\sqrt{\alpha(\alpha\zeta^2\eta^2 + \beta\delta)} \\
& + \sqrt{\alpha\gamma[4\beta\gamma \cos^2 r \sin^4 r + \alpha\gamma(1 + m - \beta \cos 2r)^2 \sin^4 r + 4\delta]} \\
& + 2\sqrt{\beta\{\alpha\beta\gamma \cos^2 r \sin^4 r + \alpha\epsilon + \beta[\cos^2 r(1 + p \sin^2 r) + n\tau \sin^2 r]^2\}} \\
& - 2\gamma[\cos^2 r(1 + p \sin^2 r) + m \sin^2 r(2 + p \sin^2 r)]\}, \tag{27}
\end{aligned}$$

where

$$\begin{aligned}
\alpha &= -1 + p, \quad \beta = -1 + m, \quad \gamma = -1 + n, \quad \tau = (2 + p \sin^2 r), \\
\delta &= (\cos r^3 + m \cos r \sin^2 r)^2, \quad \epsilon = (\cos r^3 + n \cos r \sin^2 r)^2, \\
\epsilon &= [p \cos^4 r + (m + n) \cos^2 r(1 + p \sin^2 r) + mn \sin^2 r(2 + p \sin^2 r)]^2, \\
\zeta &= \cos^2 r + m \sin^2 r, \quad \eta = \cos^2 r + n \sin^2 r. \tag{28}
\end{aligned}$$

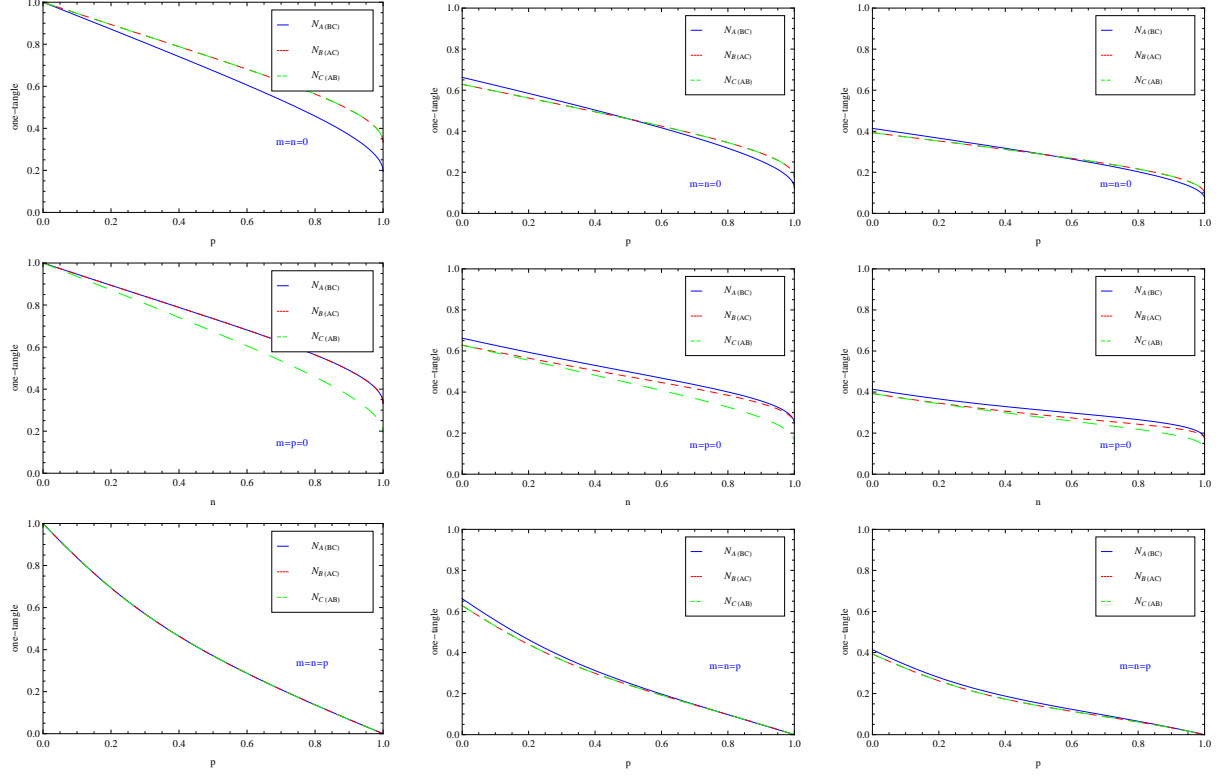


FIG. 5: (Color online) The plot shows the negativity $N_{A(B_I C_I)}$ (blue line), $N_{B_I(A C_I)}$ (red line) and $N_{C_I(A B_I)}$ (green line) for amplitude damping channel. The first (second) row presents that only inertial observer Alice (noninertial observer Charlie) is under the environment. And the third row for the case that Alice, Bob, and Charlie interact with the environment. We draw them for $r = 0$ (left rank), $r = \pi/6$ (middle rank), and $r = \pi/4$ (right rank). All the pictures have consider the normalization constant $1/\sqrt{2}$.

For one-tangle, we will study three cases: $m = n = 0$, $m = p = 0$ and $m = n = p$.

We show the one-tangles with $m = n = 0$ by the first row in Fig. 5, which means only the inertial observer Alice interacts with environment. It is very surprising to find out that the one-tangles don't vanish even with $p = 1$ which indicates that the amplitude damping channel can't destroy the tripartite entanglement no matter how longer it interacts with Alice. The three subsystems still can not be distinguished at the intersect points.

The one-tangles with $m = p = 0$ is found on the second row in Fig. 5, which means only the noninertial observer Charlie interacts with environment. If $r = 0$, $N_{B_I(A C_I)} = N_{A(B_I C_I)}$ as we expected. If $r \neq 0$, the interesting result is that Bob and Charlie have the same initial one-tangles but at last Alice and Bob have the same one-tangles, which is different from the case of *GHZ* state. That is to say, the longer the time for the environment interacting with Charlie the less the

difference between Alice and Bob is, i.e., if the time is long enough the effect of environment can wipe off the effect of acceleration even $r = \pi/4$. The tripartite entanglement doesn't vanish at $n = 1$, either.

The situation with $m = n = p$ is shown in the third row in Fig.5, which means Alice, Bob, and Charlie all are under the same environment. At $r = 0$ the three subsystems can't be distinguished as we expected. Unlike the former two cases, now we see that the interaction with environment is strong enough to destroy all the one-tangles when $m = n = p = 1$, however, no sudden death happens yet.

By use of Eq. (8) we find the two-tangle between any two subsystems of the multipartite system

$$N_{AB_I} = \frac{1}{6} \{-2 - 2m - 2p \cos 2r + 2mp \cos 2r + 2\sqrt{(1-m)(1-p)[\cos^2 r + (1-m)(1-p) \sin^4 r]} + \sqrt{4(1-m)(1-p) \cos^2 r + [1 + p + m(3+p) - (-1+m)(1+p) \cos 2r]^2 - 2mp}\}, \quad (29)$$

$$N_{AC_I} = \frac{1}{6} \{-2 - 2n - 2p \cos 2r + 2np \cos 2r + 2\sqrt{(1-n)(1-p)[\cos^2 r + (1-n)(1-p) \sin^4 r]} + \sqrt{4(1-n)(1-p) \cos^2 r + [1 + p + n(3+p) - (-1+n)(1+p) \cos 2r]^2 - 2np}\}, \quad (30)$$

$$N_{B_I C_I} = \frac{1}{12} \{4 \cos 2r - 8(m+n) \cos 2r + 12mn \cos 2r - \cos 4r + (m+n) \cos 4r - mn \cos 4r + 2\sqrt{\beta\gamma[4 \cos^4 r + \beta\gamma(-5 + \cos 2r)^2 \sin^4 r]} - 7 + 3m + 3n - 11mn + 4\sqrt{\beta\gamma \cos^4 r + [\cos^4 r - \frac{1}{2}(m+n) \cos^2 r(-3 + \cos 2r) + mn \sin^2 r(2 + \sin^2 r)]^2}\}, \quad (31)$$

here $\beta = -1 + m$, $\gamma = -1 + n$. It is easy to find that N_{AB_I} , N_{AC_I} , and $N_{B_I C_I}$ is independent on n (m , and p), i.e., a subsystem interacts with environment wouldn't affect the two-tangle between the other two subsystems. And all the initial two-tangles don't equal to zero which is different from the case of *GHZ* state.

For two-tangle, we will also study three cases: $m = n = 0$, $m = p = 0$ and $m = n = p$.

We give the results with $m = n = 0$ on the first row in Fig. 6, which indicates only Alice is under the environment. Note that $N_{B_I C_I}$ is a constant and $N_{AB_I} = N_{AC_I}$, but unlike the one-tangles they both vanish in the limit of $p = 1$, i.e., the environment can destroy the two-tangles completely just like the case of two-qubits. But the difference is that no sudden death happens yet.

We show the two-tangles with $m = p = 0$ at the second row in Fig. 6, which indicates only Charlie is under the environment. It is found that $N_{AC_I} = N_{AB_I}$ when $n = 0$ (i.e., without considering the environment). If $n > 0$, the environment will destroy the symmetry between Bob and

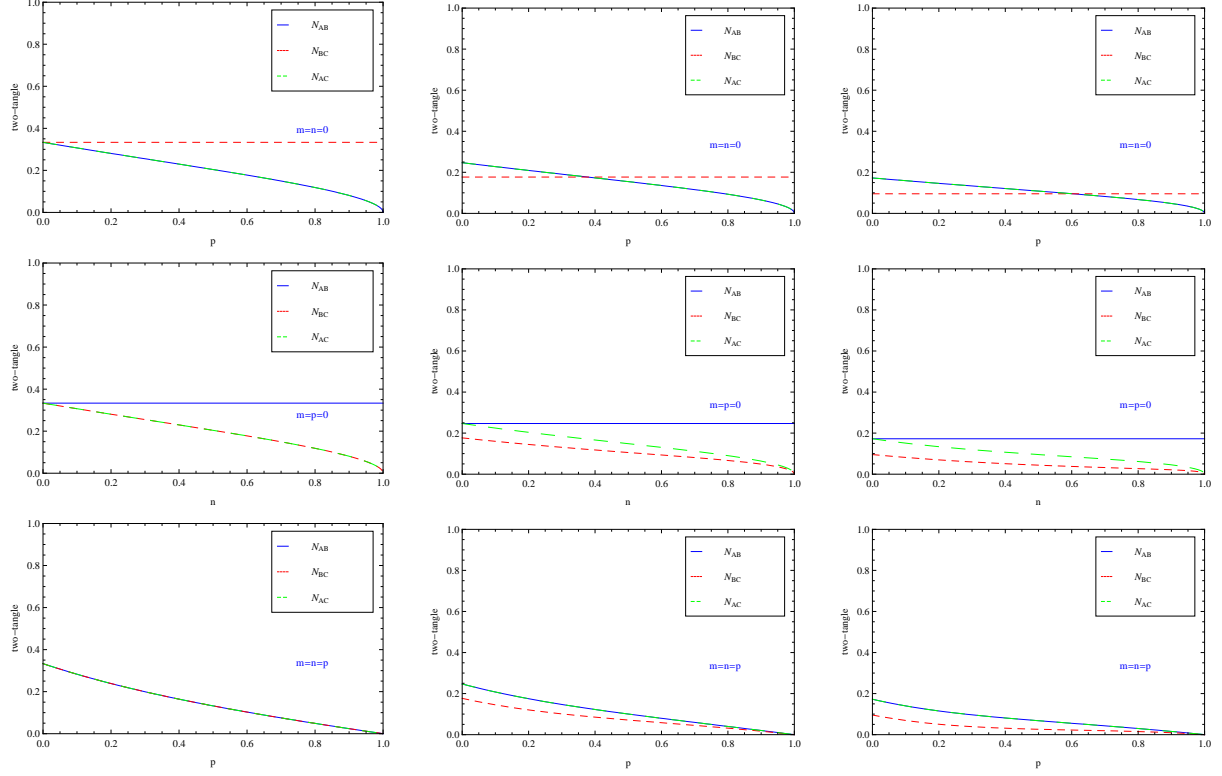


FIG. 6: (Color online) The plot shows the negativity N_{AB_I} (blue line), N_{BC_I} (red line), and N_{AC_I} (green line) for amplitude damping channel. The first (second) row presents that only inertial observer Alice (noninertial observer Charlie) is under the environment. And the third row for the case that Alice, Bob, and Charlie all interact with the environment. We draw them for $r = 0$ (left rank), $r = \pi/6$ (middle rank), and $r = \pi/4$ (right rank). All the pictures have considered the normalization constant $1/\sqrt{2}$.

Charlie as we expected, and the acceleration will destroy the symmetry between Alice and Bob. The two-tangles N_{AC_I} and $N_{B_I C_I}$ disappear completely when $n = 1$.

The two-tangles with $m = n = p$ is shown in the last row in Fig. 6, which means all the subsystems are under the environment. Now all of the two-tangles are destroyed since all of subsystems are under the environment. Note that $N_{AB_I} = N_{AC_I}$ as we expected.

At last, we compute the π -tangle by use of Eqs. (10) and (11) meanwhile consider the normalization constant $1/\sqrt{2}$. The result is found in Fig. 7, just like before, the effect of environment is much more stronger than the effect of acceleration. For W state, the initial π -tangle is smaller than that for GHZ state because there exist two-tangles.

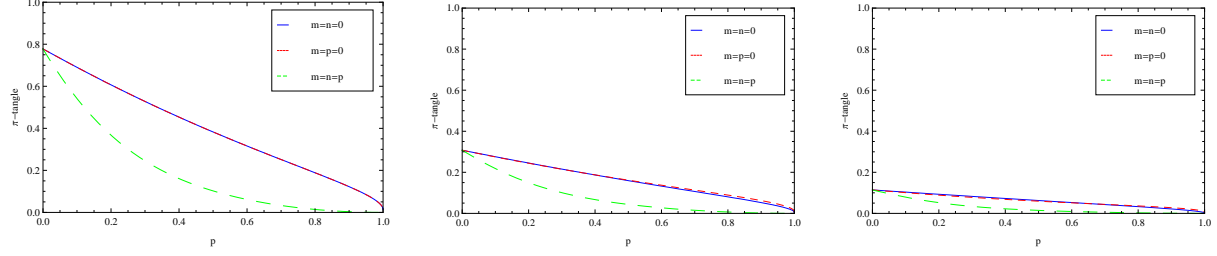


FIG. 7: (Color online) The π -angle which considers the environment. Blue (Red) line plots the case of only Alice (Charlie) interacting with the environment and green one corresponds to the case of all of them interacting with the environment. We also show three cases for $r = 0$ (left), $r = \pi/6$ (middle), and $r = \pi/4$ (right). All the pictures have considered the normalization constant $1/\sqrt{2}$.

B. Depolarizing noise

Repeating the foregoing steps and using Eqs. (2), (7) and (9), we give the results in Fig. 8 for case that all the subsystems in depolarizing noise. We see that many former characteristics still remain under this environment, too. And the rebound process is much more stronger than before when $p > 0.75$ and we can hardly ignore it any more. Similarly, the acceleration can resist the rebound process but can't destroy it. And we predict that this process would be more stronger in a higher dimensionality.

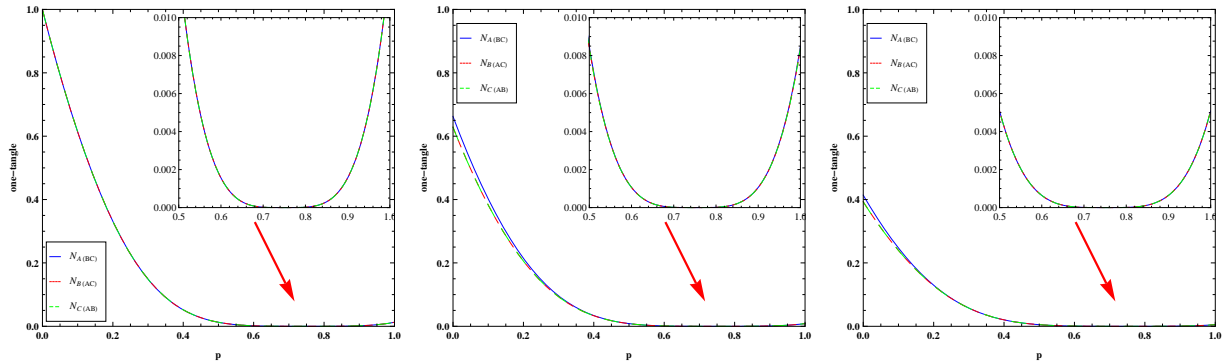


FIG. 8: (Color online) The negativity $N_{A(B|C)}$ (blue line), $N_{B(A|C)}$ (red line), and $N_{C(A|B)}$ (green line) when Alice, Bob, and Charlie all are in depolarizing noise. We show three cases for $r = 0$ (left), $r = \pi/6$ (middle), and $r = \pi/4$ (right). The rebound process is plotted in the magnifying pictures. All the pictures have considered the normalization constant $1/\sqrt{2}$.

In addition, using Eq. (8) we give $N_{AB|I}$, $N_{AC|I}$ and $N_{BC|I}$ in Fig. 9. What's surprising is that in the

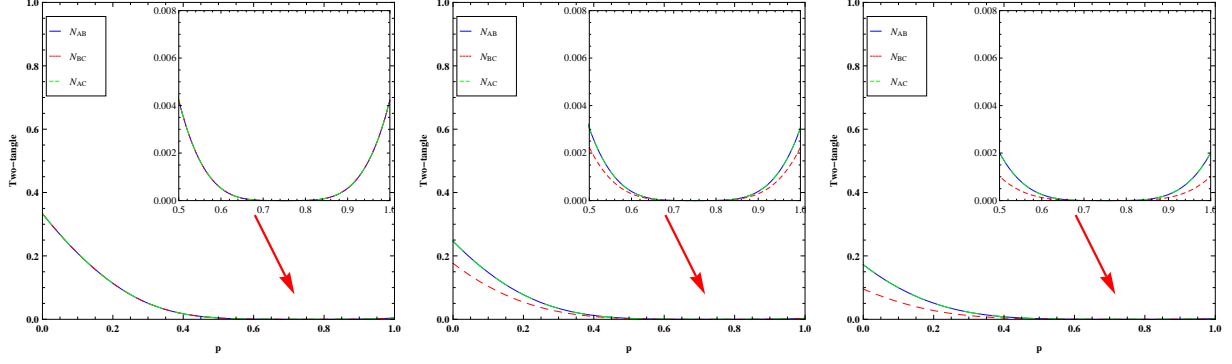


FIG. 9: (Color online) The negativity N_{AB_I} (blue line), $N_{B_I C_I}$ (red line), and N_{AC_I} (green line) when all the subsystems are under the environment. We give the cases for $r = 0$ (left), $r = \pi/6$ (middle), and $r = \pi/4$ (right). The rebound process is plotted in the magnifying pictures. All the pictures have considered the normalization constant $1/\sqrt{2}$.

tripartite system the two-tangles also have a rebound process in depolarizing noise when $p > 0.75$. The π -tangle and its rebound process are plotted in Fig. 10. Now we can say that unlike the case of two-qubits there is no sudden death even all the subsystems are in the depolarizing noise. But an entanglement rebound process appears.

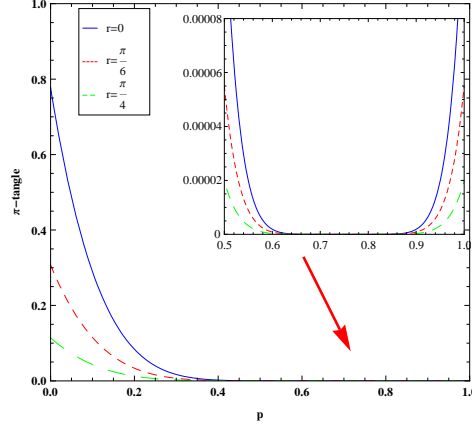


FIG. 10: (Color online) The π -tangle $\pi_{AB_I C_I}$ when Alice, Bob, and Charlie all are in depolarizing noise. We show three cases for $r = 0$ (dotted line), $r = \pi/6$ (dash line), and $r = \pi/4$ (solid line). All the pictures have considered the normalization constant $1/\sqrt{2}$.

We note again that the CKW inequality [24], $N_{AB}^2 + N_{AC}^2 \leq N_{A(BC)}^2$, is still saturated for this state, which means the effect of environment and the noninertial frames don't destroy this inequality for W initial state, either.

V. SUMMARY

The tripartite entanglement of a 3-qubit fermionic system under the amplitude damping channel and in depolarizing noise when two subsystems are accelerated for the GHZ and W initial states is investigated. It is shown that all the one-tangles and π -tangles decrease more quickly when subsystems are under environment. However, unlike the case of 2-qubit system in which sudden death can be taken place easily, here a surprising result is that no sudden death happens for any acceleration even all the subsystems are under the environment. We can't distinguish all the subsystems when $p = \cos 2r \sin^2 r$ if only Alice is under the amplitude environment for the GHZ state. All the entanglement decreases more quickly in depolarizing noise than that in amplitude damping environment. It is found that no bipartite entanglement generates either in the accelerated subsystem or under the environment for the GHZ state, i.e., all the entanglement is in form of tripartite entanglement in this case. But bipartite entanglement exists for the W state. Both the effect of acceleration and environment can destroy the symmetry between the subsystems. Thus we can perform such quantum information tasks to distinguish the accelerated-observers when some observers are accelerating by using the effect of environment or distinguish some observers in the same environment with the effect of acceleration. We furthermore give a conclusion that the more strong the subsystem interacts with the environment is, the faster the entanglement decays. And the effect of environment is so strong that we can nearly ignore the effect of acceleration if the time is long enough. In depolarizing noise environment and for both the GHZ and W initial states, the entanglement will decay to zero at $p = 0.75$ and then a rebound process takes place when $p > 0.75$, which means that all the tripartite entanglement transfers to environment at $p = 0.75$ and then part of it transfers from environment back to the system when $p > 0.75$. The CKW inequality $N_{AB}^2 + N_{AC}^2 \leq N_{A(BC)}^2$ is saturated for any case in this paper, which means the effects of environment and acceleration don't destroy this inequality.

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